Rotating shallow water flow under location uncertainty

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Part II: some numerical results

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Governing equations

- Conservation of momentum

$$\mathrm{D}_t \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} \, \mathrm{d}t = -g \boldsymbol{\nabla} h \, \mathrm{d}t \tag{1}$$

Stochastic transport operator: $\mathrm{D}_t[\bullet] = \left(\mathrm{d}_t + (u - \boldsymbol{\nabla}\cdot\frac{1}{2}\boldsymbol{a})\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t\right)\cdot\boldsymbol{\nabla}\,[\bullet] - \boldsymbol{\nabla}\cdot\left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}[\bullet]\right)$

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- Conservation of mass

$$\mathrm{D}_t h + h \, \boldsymbol{\nabla} \cdot \boldsymbol{u} \, \mathrm{d}t = 0 \tag{2}$$

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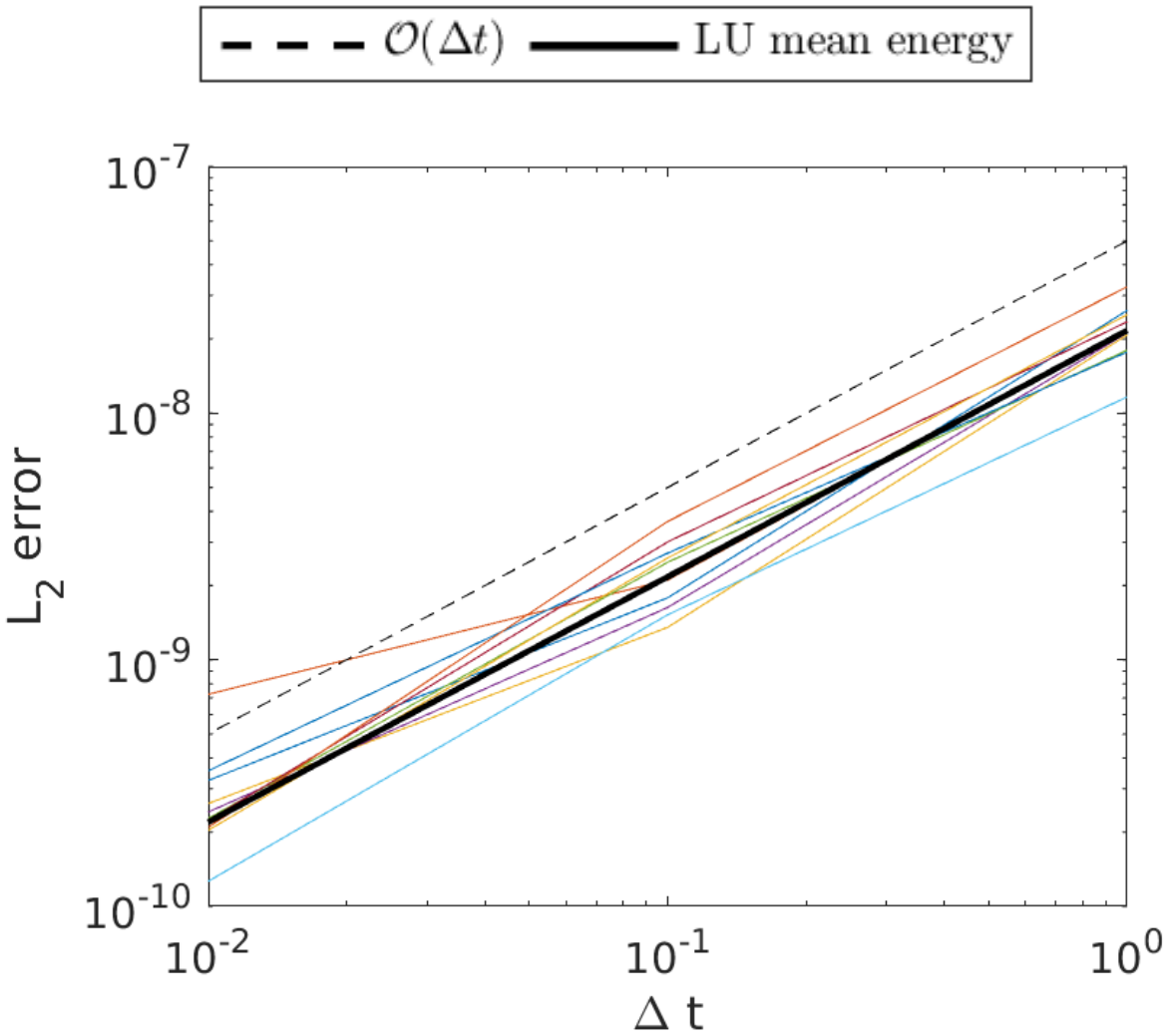
- Conservation of energy

$$\mathrm{d}_t \int_\Omega \frac{\rho}{2}\left(h|\boldsymbol{u}|^2 + gh^2\right)\mathrm{d}\boldsymbol{x} = 0 \tag{4}$$

Stochastic transport operator: $\mathrm{D}_t[\bullet] = \left(\mathrm{d}_t + (u - \boldsymbol{\nabla}\cdot\frac{1}{2}\boldsymbol{a})\mathrm{d}t + \boldsymbol{\sigma}\mathrm{d}\boldsymbol{B}_t\right)\cdot\boldsymbol{\nabla}\,[\bullet] - \boldsymbol{\nabla}\cdot\left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}[\bullet]\right)$

Energy diagnosis



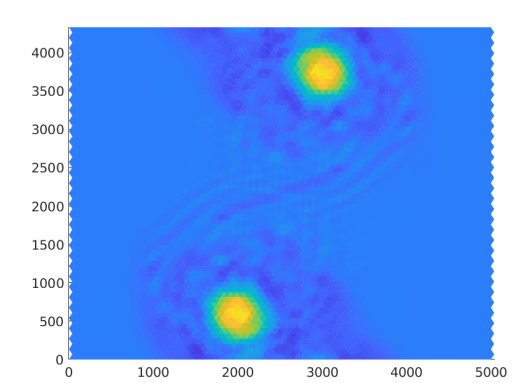
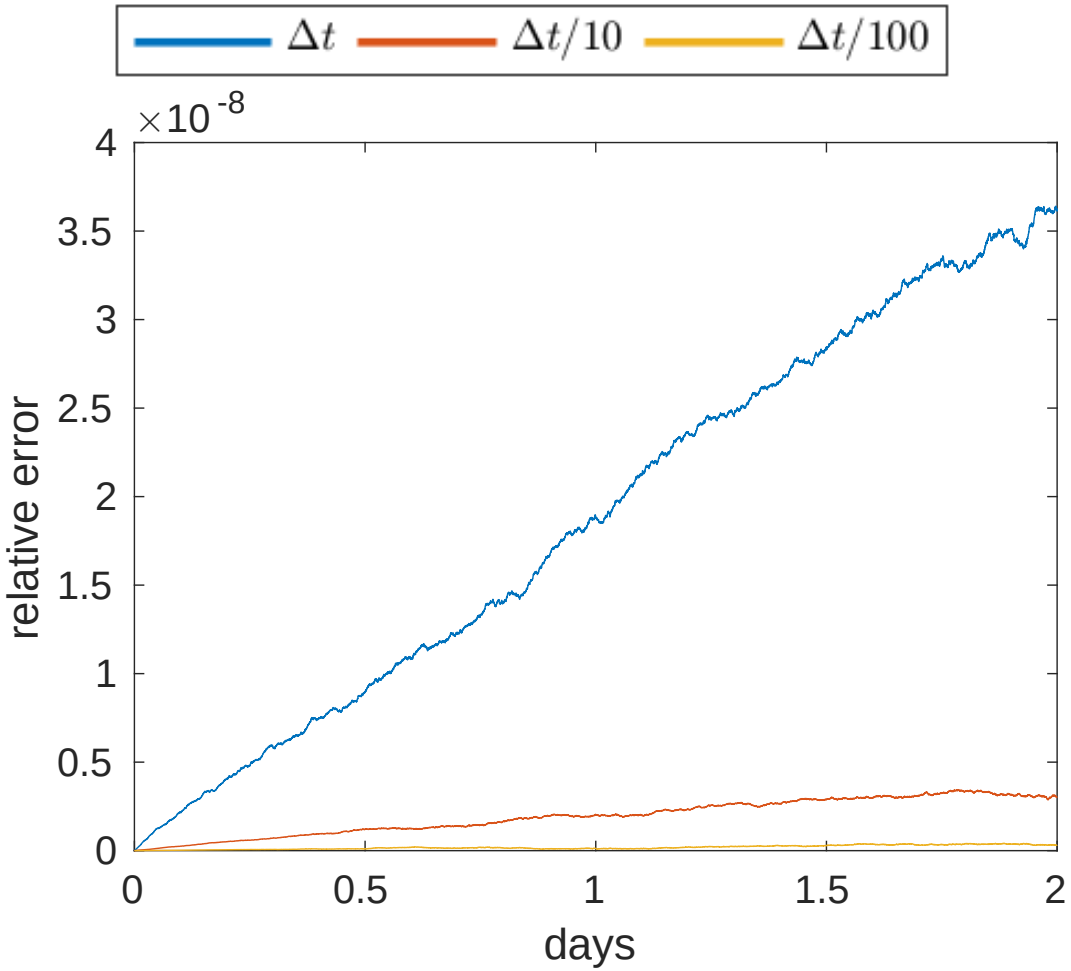


Figure: Convergence in temporal resolution of LU ensemble energy to the reference (at spatial resolution 128^2).

Energy diagnosis



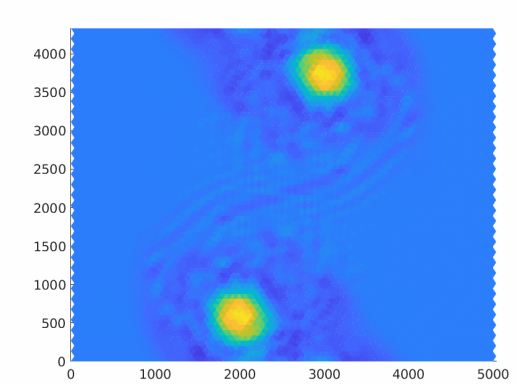
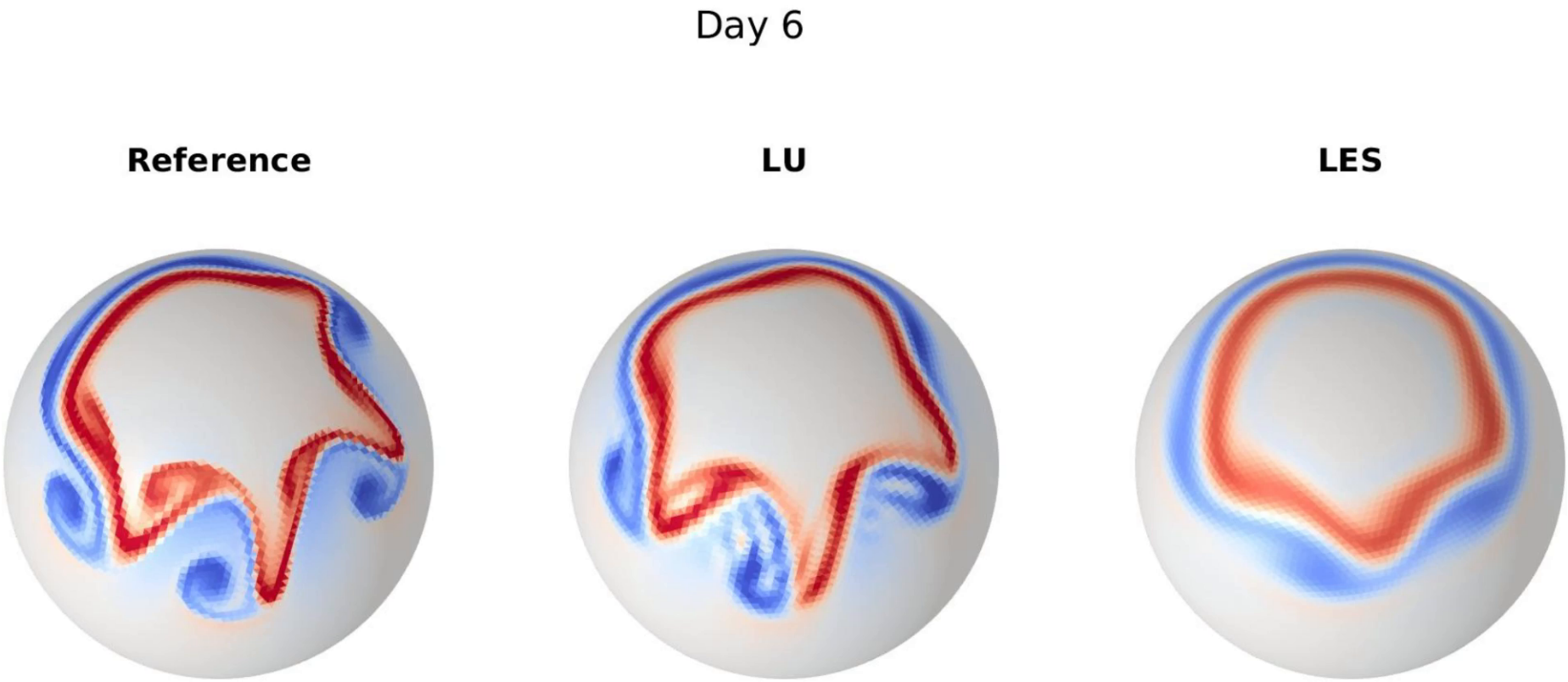


Figure: Relative errors of LU ensemble mean energy compared to the reference (at spatial resolution 128^2).

Barotropic instability



Video: Evolution of vorticity fields.

Barotropic instability

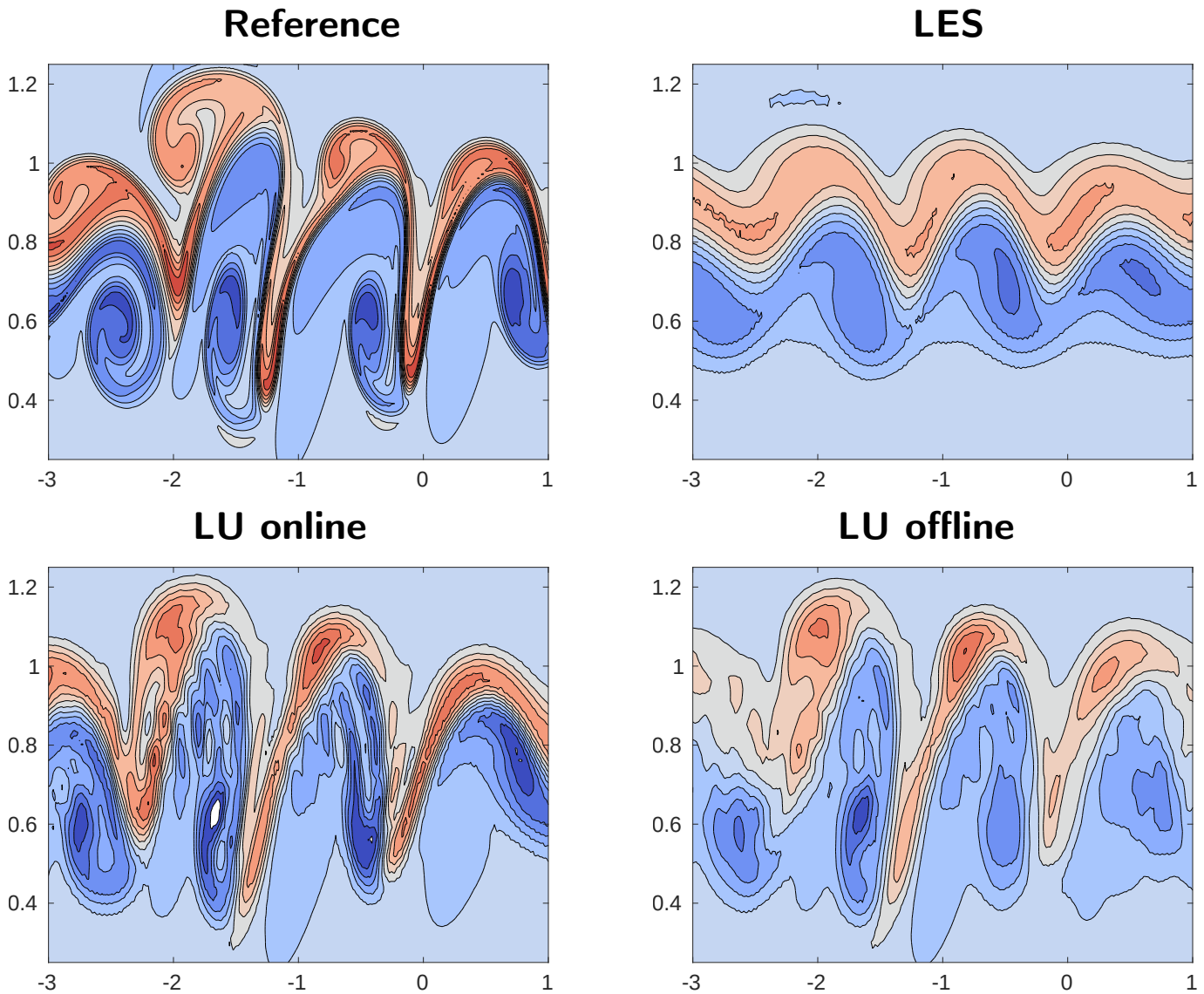


Figure: Contour snapshots of vorticity field at day 6 with CI $= 10^{-5}\ s^{-1}$.

Barotropic instability

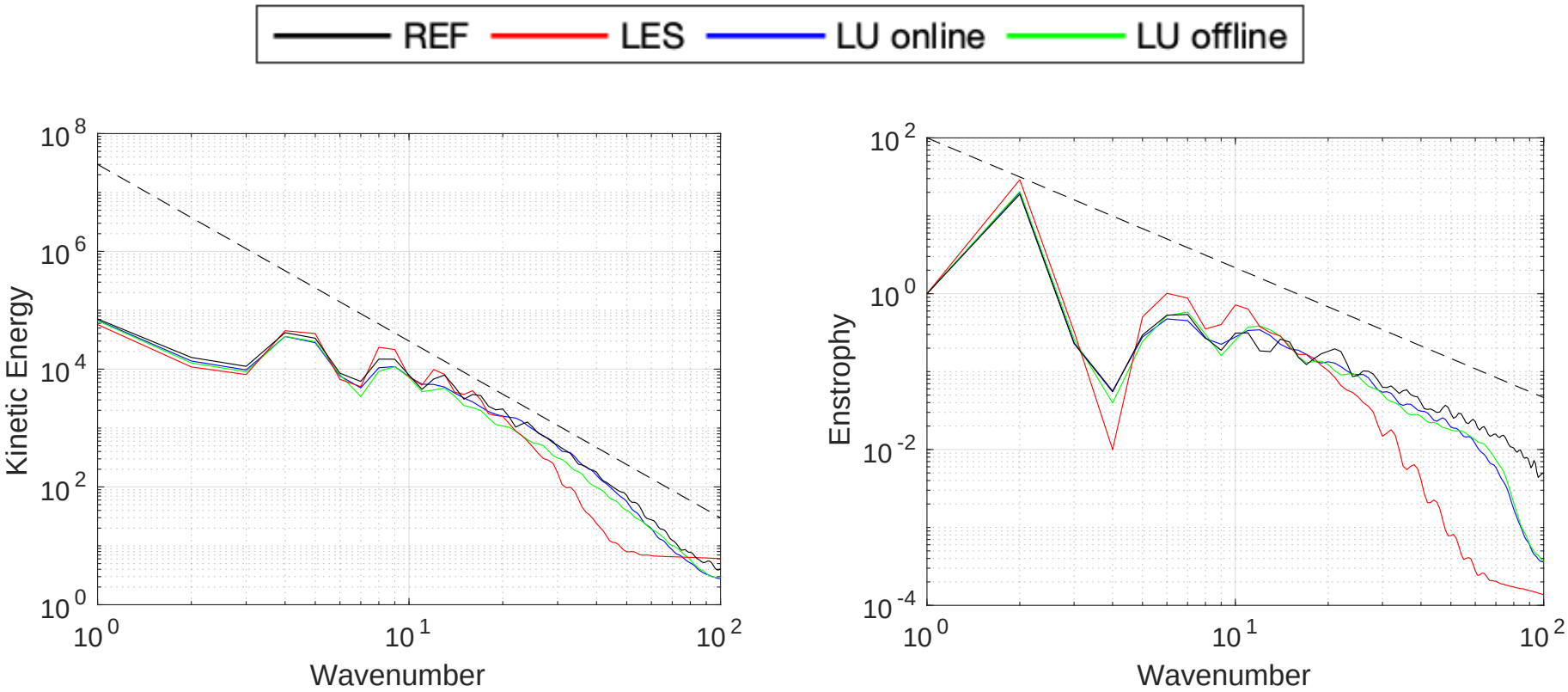
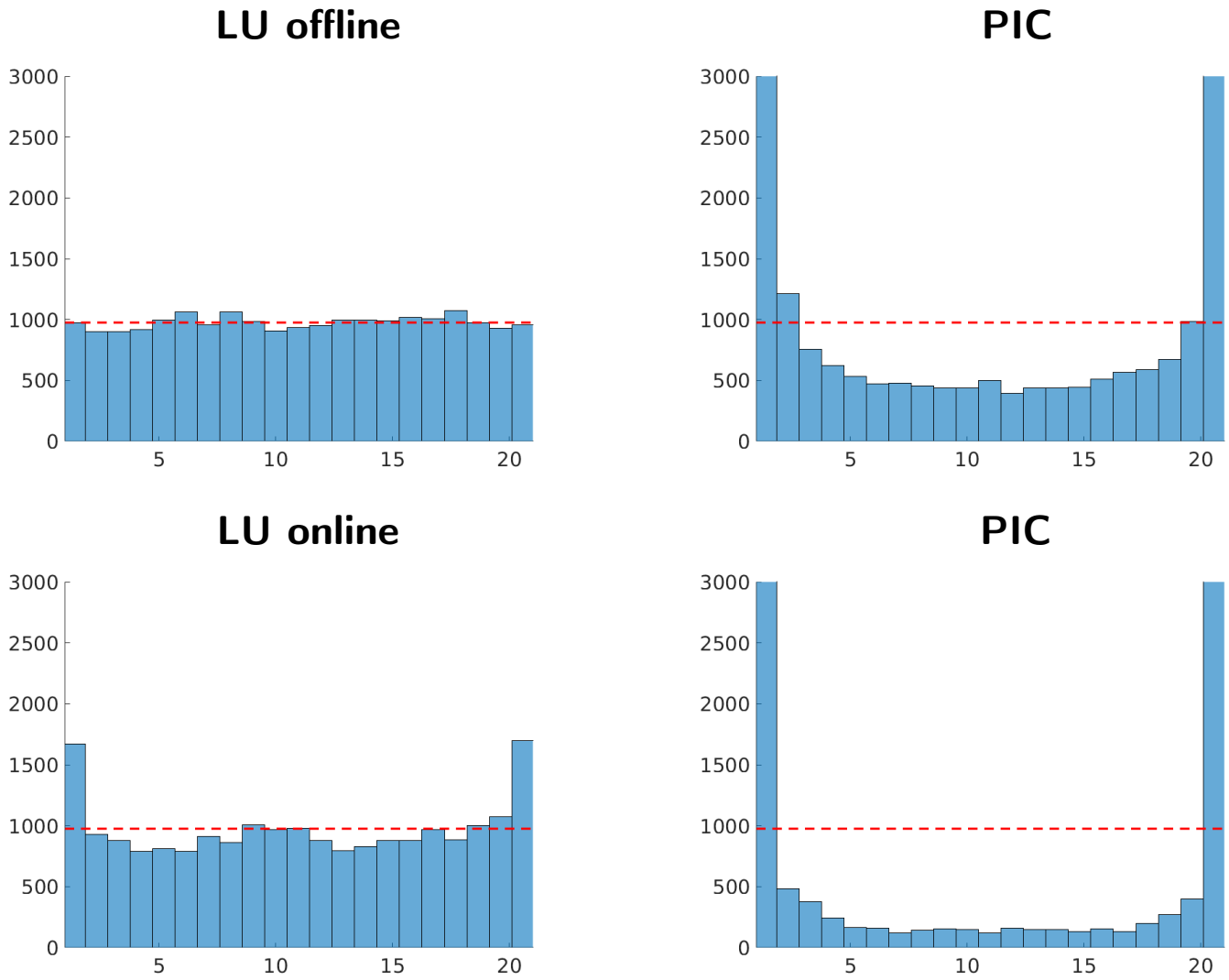


Figure: Spectrums of kinetic energy (left) and enstrophy (right) at day 6. The dashed lines are power laws of slope -3 (left) and $-5/3$ (right).

Ensemble forecast



Video: Evolution of rank histograms from day 1 to day 20.

Thank for Your Attention!